Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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# MONETARY POLICY
# SUGGESTED SOLUTIONS TO JUNE 5 EXAM, 2018

## QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model, the central bank should refrain from policies that affect inflation expectations as this will worsen the inflation-output gap trade off.

A **False.** The ability to affect inflation expectations creates an additional channel for affecting inflation besides the change in the output gap caused by interest rate changes. The central bank can thus obtain a more favorable inflation-output gap trade off, as it can reduce inflation at a lower reduction in the output gap, if it can affect inflation expectations downwards. This, however, requires credibility of a commitment to some path of policies. Under discretionary policy, such commitment is absent, and the New Keynesian model thus provides an example of benefits from commitment.

(ii) According to the Friedman rule, the optimal rate of inflation is zero.

A **False**. The Friedman rule stipulates that the private opportunity costs of holding real money balances should be zero. I.e., the nominal interest rate should be zero. According to the Fisher relationship, this implies an inflation rate equal to the negative of the real rate of return (i.e., deflation equal to the real interest rate).

(iii) Under a nominal interest-rate operating procedure, monetary policymaking performed without knowledge of the realizations of current shocks can be improved by using money stock data as an intermediate target whenever money-market shocks are predominant in the economy.

A **False.** When money-market shocks are predominant, movements in the observable money stock will be relatively uninformative about shocks that affect output and inflation. Hence, adjusting the interest rate in response to money-stock movements will not improve monetary policy.

## QUESTION 2:

Consider an economy formulated in discrete time, where the utility of a representative agent is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1, \tag{1}$$

where $c_t$ is real consumption and $u' > 0$, $u'' < 0$. The agent faces the budget constraint

$$\begin{aligned} \omega_t &\equiv f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{m_{t-1} + (1+i_{t-1}) b_{t-1}}{1+\pi_t} \\ &= c_t + k_t + m_t + b_t, \end{aligned} \tag{2}$$

where $k_{t-1}$ is real capital at the end of period $t-1$, $f$ is a production function with $f' > 0$, $f'' < 0$, $\tau_t$ denotes real monetary transfers from the government, $0 < \delta < 1$ is the rate of depreciation of capital, $m_{t-1}$ denotes real money holdings at the end of period $t-1$, $i_{t-1}$ is the nominal interest rate on bonds (denoted $b_{t-1}$ in real terms), and $\pi_t$ is the rate of inflation.

The agent also faces a cash-in-advance constraint on consumption:

$$c_t \leq \frac{m_{t-1}}{1+\pi_t} + \tau_t. \tag{3}$$

(i) Characterize the optimal choices of consumption, capital and real money holdings. For that purpose use that the agent's optimization problem can be stated as

$$V(\omega_t, m_{t-1}) = \max_{c_t, k_t, m_t} \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left( c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

where $\mu_t$ is the multiplier on (3) and

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t (\omega_t - c_t - k_t - m_t), \quad R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}.$$

Then derive and interpret these optimality conditions:

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t,$$

$$\beta V_\omega(\omega_{t+1}, m_t)\left[f_k(k_t) + 1 - \delta\right] = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Finally, show that by use of the envelope theorem one finds

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t),$$

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}.$$

A Using that (2) is forwarded to

$$\omega_{t+1} = f(k_t) + \tau_{t+1} + (1-\delta) k_t + \frac{m_t}{1+\pi_{t+1}} + R_t(\omega_t - c_t - k_t - m_t),$$

and can be inserted into Bellman equation,

$$V(\omega_t, m_{t-1}) = \max_{c_t, k_t, m_t} \left\{ u(c_t) + \beta V(\omega_{t+1}, m_t) - \mu_t \left( c_t - \frac{m_{t-1}}{1+\pi_t} - \tau_t \right) \right\},$$

one can find the first-order condition with respect to $c_t$ as (noting that $\partial\omega_{t+1}/\partial c_t = -R_t$)

$$u_c(c_t) - \beta R_t V_\omega(\omega_{t+1}, m_t) - \mu_t = 0,$$

from which one readily recovers the desired

$$u_c(c_t) = \beta R_t V_\omega(\omega_{t+1}, m_t) + \mu_t.$$

In optimum, the agent chooses consumption at $t$ such that the marginal gain in terms of marginal utility equals the marginal loss, which takes the form of the utility loss arising from less wealth in the next period (multiplied by the real interest rate and discounted back to period $t$ by $\beta$) as well as the loss accruing from the CIA constraint.
The first-order condition with respect to $k_t$ is

$$\beta V_\omega(\omega_{t+1}, m_t)\left[f_k(k_t) + 1 - \delta\right] - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0,$$

which is readily rewritten as required:

$$\beta V_\omega(\omega_{t+1}, m_t)\left[f_k(k_t) + 1 - \delta\right] = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Capital is chosen such that the associated marginal gain in terms of more wealth in next period (multiplied by the net marginal product of capital), equals the marginal loss in terms of lower wealth in form of bonds (multiplied by the real interest rate). Note that this expression delivers the familiar relationship $R_t = f_k(k_t) + 1 - \delta$ in this case where capital and bonds are perfect substitues.
The first-order condition with respect to $m_t$ is

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) - \beta R_t V_\omega(\omega_{t+1}, m_t) = 0$$

which is readily rewritten as needed:

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Real money holdings are chosen such that the marginal gains (in terms of more wealth in the next period as well as more value of money *per se* if $V_m(\omega_{t+1}, m_t) > 0$), equal the marginal loss in terms of the value loss of lower interest-bearing wealth.

Differentiating the value function with respect to $\omega_t$ and taking into account that $c_t$, $k_t$ and $m_t$ will be optimal functions of the states ($\omega_t$ and $m_{t-1}$) whereby one can ignore any effects of $\omega_t$ on those variables, one immediately obtains

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

Likewise, differentiating the value function with respect to $m_{t-1}$, ignoring the effects on $c_t$, $k_t$ and $m_t$, one gets the desired

$$V_m(\omega_t, m_{t-1}) = \mu_t \frac{1}{1+\pi_t}.$$

Here one sees that the marginal value of money is only positive if the CIA constraint binds; i.e., in which case money provides liquidity services.

(ii) Define $\lambda_t \equiv V_\omega(\omega_t, m_{t-1})$, and derive the expression for the nominal interest rate, $i_t$, as a function of $\mu_{t+1}$ and $\lambda_{t+1}$. Explain this relationship with particular focus on the role of a binding or non-binding cash-in-advance constraint.

A The money demand function, restated here,

$$\beta \frac{1}{1+\pi_{t+1}} V_\omega(\omega_{t+1}, m_t) + \beta V_m(\omega_{t+1}, m_t) = \beta R_t V_\omega(\omega_{t+1}, m_t)$$

can with the expression for the value function derivative $V_m$ found in (i) and the definition of $\lambda_t$, be expressed as

$$\frac{1}{1+\pi_{t+1}} \lambda_{t+1} + \mu_{t+1} \frac{1}{1+\pi_{t+1}} = R_t \lambda_{t+1}.$$

This yields

$$\lambda_{t+1} + \mu_{t+1} = R_t (1+\pi_{t+1}) \lambda_{t+1},$$

and thus

$$\lambda_{t+1} + \mu_{t+1} = (1+i_t) \lambda_{t+1},$$

by the Fisher equation. Further, we get

$$\frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_{t+1}} = (1+i_t),$$

and thus the needed expression for $i_t$:

$$i_t = \frac{\mu_{t+1}}{\lambda_{t+1}}.$$

It is seen that the nominal interest rate is only positive if the CIA constraint binds. In that case, money is necessary to purchase goods, and the price of goods is increased by the opportunity cost of holding that money, and that is indeed when the nominal interest rate is positive. A brilliant answer will reconsider the first-order condition for consumption, which with the definitions of $\lambda_t$ and $\mu_t$ takes the form

$$u_c(c_t) = \beta R_t \lambda_{t+1} + \mu_t$$

Since the first result of (i) gives $\lambda_t = \beta R_t \lambda_{t+1}$ this becomes

$$\begin{aligned} u_c(c_t) &= \lambda_t + \mu_t \\ &= \lambda_t \left(1 + \frac{\mu_t}{\lambda_t}\right) \\ &= \lambda_t (1 + i_{t-1}). \end{aligned}$$

This clearly shows that with a binding CIA constraint, and thus positive nominal interest rate, consumption is being "taxed" by the constraint.

(iii) Show formally that monetary policy—different rates of nominal money growth—has no real effects in the steady state of this economy. Explain the result. Which variables will, on the other hand, be affected by different long-run nominal money growth rates? Explain.

A From the expression

$$V_\omega(\omega_t, m_{t-1}) = \beta R_t V_\omega(\omega_{t+1}, m_t).$$

one immediately recovers the steady-state relationship

$$R^{ss} = \frac{1}{\beta}$$

Combining this with the expression for the real interest rate, one gets

$$\frac{1}{\beta} = f_k(k^{ss}) + 1 - \delta$$

Hence, the capital stock is determined unrelated to monetary factors. The reason is that the capital accumulation process in the model is not distorted by the CIA constraint. The steady-state real interest rate is given by the households' subjective real interest rate $(1/\beta)$, and that is exclusively given by the net marginal product of capital. From the national account identity, $c^{ss} = f(k^{ss}) - \delta k^{ss}$, it thus follows that consumption is unaffected as well. Different long-run monetary growth rates will therefore only affect monetary factors. Higher money growth will lead to higher inflation, and thus, as the real interest rate is constant, to higher nominal interest rates.

Despite a higher nominal interest rate, real money holdings do not change as agents only hold money to cover consumption purchases. Showing this using the public budget constraint would be excellent. From the binding CIA constraint one gets

$$c_t = \frac{m_{t-1}}{1+\pi_t} + \tau_t.$$

Taking into account that nominal transfers are financed by money creation, $P_t \tau_t = M_t - M_{t-1}$ where $P_t$ is the price level and $M_t$ is the nominal money stock, one has real transfers given as

$$\begin{aligned} \tau_t &= \frac{M_t - M_{t-1}}{P_t} = m_t - \frac{M_{t-1}}{P_t} = m_t - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} \\ &= m_t - \frac{m_{t-1}}{1+\pi_t}, \end{aligned}$$

which inserted into the CIA constraint gives

$$\begin{aligned} c_t &= \frac{m_{t-1}}{1+\pi_t} + m_t - \frac{m_{t-1}}{1+\pi_t} \\ &= m_t. \end{aligned}$$

So, in steady state, $c^{ss} = m^{ss}$. Hence, the real money supply does not change with the nominal interest rate or inflation.

## QUESTION 3:

Consider the following simple New-Keynesian model of a closed economy:

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - (i_t - \mathrm{E}_t \pi_{t+1} - r) + u_t, && (1) \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, && (2) \\ i_t &= r + \delta \pi_t, \qquad \delta > 1, && (3) \end{aligned}$$

where $x_t$ is the output gap, $i_t$ is the nominal interest rate, $r$ is the steady-state real interest rate, $\pi_t$ is goods-price inflation, and $u_t$ is a mean-zero i.i.d. shock. $\mathrm{E}_t$ is the rational-expectations operator conditional upon all information up to and including period $t$.

(i) Describe in detail how equations (1) and (2) arise from optimal decisions by representative agents in the economy. Discuss briefly why $\delta > 1$ is assumed.

A Equation (1) is the "dynamic IS curve", which as a foundation uses a log-linearization of consumers' consumption-Euler equations: A lower real interest rate, $i_t-$ $\mathrm{E}_t\pi_{t+1}$, make consumers increase current consumption relative to future consumption. With consumption, $c_t$, being equal to output, $y_t$, application of the definition of the output gap as the difference between output at flex-price output, leads to (1). The shock $u_t$ will then represent expected changes in the flex-price output. It can be noted that fluctuations in $u_t$ represents fluctuations in the natural rate of interest.

Equation (2), the "New-Keynesian Phillips Curve", is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness of the Calvo form. Prices are set as a mark up over marginal costs, and as the output gap is proportional to marginal costs, it enters in (2) positively. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be in effect for some periods.

Equation (3) is a characterization of interest-rate policy. The nominal interest rate is set above the steady-state real interest rate $r$ whenever inflation is higher than the zero steady state. Vice versa when inflation is lower than steady state. With $\delta > 1$ the interest-rate response to inflation changes are sufficiently strong to impact the *real* interest rate. This is a desirable property in this type of model, as it secures uniqueness of a non-explosive equilibrium ("determinacy").

(ii) Assume that stabilizing the output gap, $x_t$ and $\pi_t$ is preferable. Discuss why this is a reasonable assumption often made in this type of model.

A In this model, the output gap is proportional to the real marginal costs of producers. Nominal rigidities causes fluctuations in real marginal costs and thus employment and consumption which are undesirable. A stable output gap is synonymous with stable real marginal costs, which eliminates this distortion of nominal rigidities. Moreover, nominal rigidities imply that any inflation different from zero induces relative price changes and thus inefficient demand dispersion of the various consumer goods in the economy.

(iii) Evaluate formally whether stabilizing $x_t$ and $\pi_t$ perfectly *at the same time,* is possible in the model by appropriate choice of $\delta$. Explain. [Hint: Conjecture that the solutions for $x_t$ and $\pi_t$ are linear functions of $u_t$, and use the method of undetermined coefficients.]. Discuss the associated solution for the nominal interest rate.

A Use the hint and make the following conjectures:

$$x_t = Au_t, \qquad \pi_t = Bu_t.$$

Then forward the conjectures one period and take expectations:

$$\mathrm{E}_t x_{t+1} = A\mathrm{E}_t u_{t+1}, \qquad \mathrm{E}_t \pi_{t+1} = B\mathrm{E}_t u_{t+1}.$$

Since the shock is mean zero and i.i.d., we readily obtain

$$\mathrm{E}_t x_{t+1} = 0, \qquad \mathrm{E}_t \pi_{t+1} = 0.$$

Then insert the conjectures, and their expectations, into the model where the interest rate in (1) is substituted out by the rule in (3):

$$\begin{aligned} x_t &= \mathrm{E}_t x_{t+1} - (\delta\pi_t - \mathrm{E}_t \pi_{t+1}) + u_t, \\ \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \end{aligned}$$

leading to

$$\begin{aligned} Au_t &= -\delta B u_t + u_t, \\ Bu_t &= A\kappa u_t. \end{aligned}$$

This verifies the form of the conjecture, and requires that the unknown coefficients satisfy

$$\begin{aligned} A &= -\delta B + 1, \\ B &= A\kappa. \end{aligned}$$

Therefore we get

$$\begin{aligned} B &= (-\delta B + 1)\,\kappa \\ B &= \frac{\kappa}{1+\kappa\delta} \end{aligned}$$

and

$$A = \frac{1}{1+\kappa\delta}.$$

The solutions for the output gap and inflation are therefore

$$\begin{aligned} x_t &= \frac{1}{1+\kappa\delta} u_t, \\ \pi_t &= \frac{\kappa}{1+\kappa\delta} u_t. \end{aligned}$$

From the solutions we see that full stabilization of the output gap and inflation at the same time *is* possible (the "divine coincidence" applies). If $\delta \to \infty$, then $x_t \to 0$ and $\pi_t \to 0$ at the same time. The nominal interest rate in this limit will become as follows. First note that the solution of the nominal interest rate is

$$i_t = r + \delta \frac{\kappa}{1+\kappa\delta} u_t.$$

Then, in the limit of $\delta \to \infty$ we have

$$\begin{aligned} i_t &= r + \lim_{\delta \to \infty} \frac{\kappa\delta}{1+\kappa\delta} u_t, \\ &= r + u_t. \end{aligned}$$

where the second line follows from l'Hôpital's rule. It is seen that the nominal interest rate moves with the natural rate of interest.